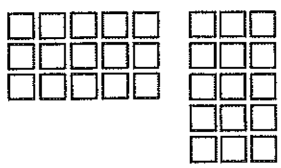
Math Background

Concept Building Activities

Groups, Arrays, and Area The concepts of multiplication as a fast way to count groups and as a way to find the total in arrays were introduced in Unit 1. By turning an array 90°, the students discovered commutativity (for example, $5 \times 3 = 3 \times 5$). In geometry Unit A, the students used arrays of pushed-together squares to represent areas of rectangles.

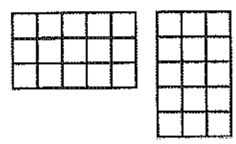
Arrays of Squares



3 rows of 5 each *or* 5 rows of 3 each

3×5 or

Arrays of Pushed-Together Squares

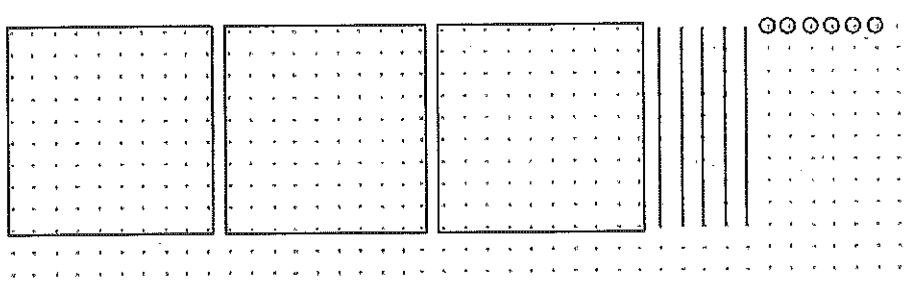


5 rows of 3 each *or* 3 rows of 5 each

5×3

This unit builds on the concept of area as pushed-together arrays of squares to help students understand multi-digit multiplication.

Place-Value Models Lessons on multi-digit addition and subtraction in Unit 2 used the MathBoard. These activities provided tangible, graphic support for the students' understanding of the relative sizes of the ones, tens, hundreds, and higher places. The students drew and described place-value diagrams of numbers, such as the one shown below for 356.



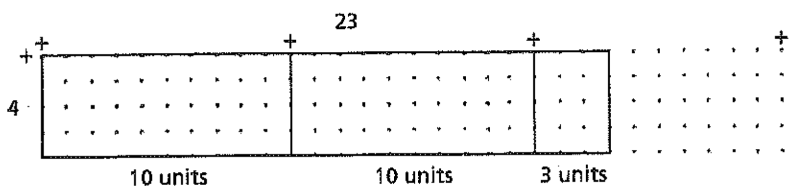
3 boxes that each contain 100 dots = 3 hundreds = 300

5 segments that each connect 10 dots = 5 tens = 50

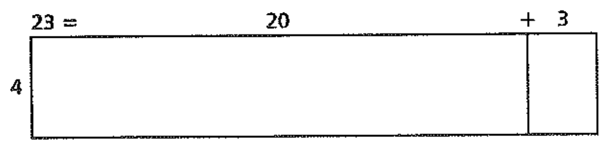
6 circled single dots = 6 ones = 6

This activity helped the students understand that a 3 in the hundreds place is not just 3, but actually represents 300. It also helped prepare the way for the understanding of place value needed to conceptualize multi-digit multiplication.

Area Models In using place-value drawings, the students counted the dots. In Unit 3, students will be finding the number of unit "spaces" between the dots in order to find total areas of rectangles. Little crosses (+) every 10 spaces horizontally and vertically make it easier to count the units. The drawing below shows such a representation for the multiplication 23 × 4.



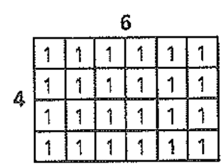
As students progress through the unit, usually they will be able to draw rectangles without using the dots. On the MathBoard, they should move from the side with the dots to the side without the dots. Their understanding will be approaching a higher level of abstraction.



Multiplication Models Any multiplication can be drawn as a rectangle.

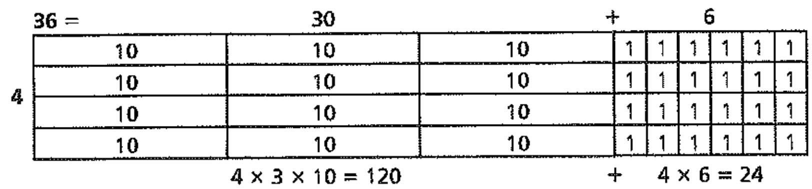
Single-digit times single-digit

The drawing at the right shows 4 × 6 = 24. It is a 4-by-6 array of ones that shows ones-by-ones place multiplication.



Single-digit times multi-digit

The drawing below shows 4 × 36 = 144.



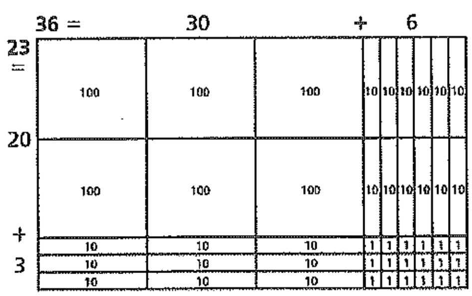
The rectangle is made up of two arrays:

- A 4-by-3 array of groups of ten that shows the ones-by-tens place multiplication: 4 × 3 × 10 = 120
- A 4-by-6 array of ones that shows the ones-by-ones place multiplication: 4 × 6 = 24

Adding the tens group product and the ones product gives the total: 144

Multi-digit times multi-digit

The drawing below shows 23 × 36 = 828.



The rectangle is made up of four arrays:

- A 2-by-3 array of hundreds groups that shows the tens-by-tens place multiplication: 2 × 3 × 100 = 600
- A 3-by-3 array of groups of tens that shows the ones-by-tens place multiplication: 3 × 3 × 10 = 90
- A 2-by-6 array of groups of ten that shows the tens-by-ones place multiplication: 2 × 6 × 10 = 120
- A 3-by-6 array of ones that shows the ones-by-ones place multiplication: 3 × 6 = 18

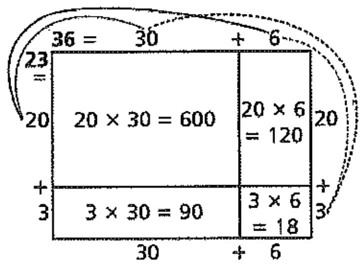
Adding the hundreds, tens, and ones groups gives the total: 828

Multiplication Methods The method of drawing a rectangle and adding the area of all the sections is referred to as the **Rectangle Sections Method** in this unit.

In all numeric multiplication methods, each single digit in one number must be multiplied by each single digit in the other number.

For multi-digit multiplication, area models with connecting arcs can help students keep track of which numbers they need to multiply. This area model is related to the following numeric method. The colored arcs in the multiplication correspond to the like-colored arcs in the model.

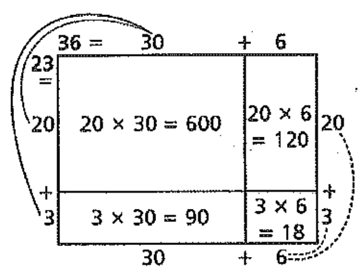
$$\begin{array}{rr} 36 = & 30 + 6 \\ \times\ 23 = & 20 + 3 \\ \hline & 20 \times 30 = 600 \\ & 20 \times 6 = 120 \\ & 3 \times 30 = 90 \\ & 3 \times 6 = 18 \\ \hline & = 828 \end{array}$$



This method is referred to as the **Expanded Notation Method** in this unit.

Another numeric method, called the **Algebraic Notation Method**, is closely related to the Expanded Notation Method because students must multiply each digit in one number by each digit in the other number.

$$36 \cdot 23 = (30 + 6) \cdot (20 + 3)$$
$$= 600 + 90 + 120 + 18$$
$$= 828$$



The discussion of numeric methods concludes with the **Shortcut Method**, the common algorithm taught in most U.S. schools. Not all students are expected to gain proficiency using this method in this unit. Rather, it is preferred that students are capable of understanding multiplication using one of the methods above.

$$\begin{array}{r} {}^{1}{}_{1} \\ 36 \\ \times\ 23 \\ \hline 108 \\ 72\ \\ \hline 828 \end{array}$$

Patterns in Products In this unit, students look at patterns of products in which one of the numbers multiplied is a ten or a power of ten (such as 10, 100, and 1,000). Students examine groups of related multiplications such as:

$6 \times 3 = 6 \times 3 \times 1 = 18$

$6 \times 30 = 6 \times 3 \times 10 = 18 \times 10 = 180$

$6 \times 300 = 6 \times 3 \times 100 = 18 \times 100 = 1{,}800$

They rewrite each product so one factor is a power of ten. Then they see that the product is the product of the non-zero digits (in this case, 6×3), followed by the number of zeros in the power of 10.

Area models help students understand *why* they can "count the zeros" when multiplying by tens and tens groups. These patterns can also help students understand how to estimate products and round and multiply to check the accuracy of their calculations.

Problem Solving

Real-World Problem Solving

Throughout the unit, real-world situations are used as the context for problem-solving situations, including problems that involve two or more different operations. Students are also expected to analyze problems to determine whether they have too little or too much information to solve. Students are encouraged to make mathematical connections to their content-area subjects, particularly, social studies and science, as well as in their lives at home and school. Teachers are encouraged to talk about real-world math applications to help students attend to how they and others use mathematics in everyday life.

This program's strong and continuous focus on solving a wide range of word problems helps with language development. The Math Talk approach to discussing word problems facilitates language growth. Students have experienced substantial increases in vocabulary and in reasoning; such gains can be beneficial in areas other than math.

This systematic and research-based focus on word problems has changed the word problem solving modes of many students: They no longer just look at the numbers and do something with them (for example, add or multiply all numbers they see). They think about the situation and try to understand it.